Chapter 15 Probability Rules!

Tree Diagrams

Suppose that 47% of Yorktown's population is male. Of the male population, 87% enroll in a 4-year college. Of the female population, 93% enroll in a 4-year college.

a) Draw a tree diagram:

\[
\begin{align*}
\text{male} & : 0.47 & \text{enroll in 4-year college} & : 0.41 \\
& & & \text{don't enroll in 4-year college} & : 0.47 \\
\text{female} & : 0.53 & \text{enroll in 4-year college} & : 0.93 \\
& & & \text{don't enroll in 4-year college} & : 0.07
\end{align*}
\]

b) What is the probability that a randomly selected student will go to college?

\[
P(\text{college}) = P(\text{m} \cap \text{college}) + P(\text{f} \cap \text{college}) = 0.41 + 0.48 = 0.89
\]

c) What is the probability that a Yorktown student who goes to college is male?

\[
P(\text{m} | \text{college}) = \frac{P(\text{m} \cap \text{college})}{P(\text{college})} = \frac{0.41}{0.89} = 0.465
\]

d) Given that a student has selected not to go to college, what is the probability that this student is female?

\[
P(\text{f} \mid \text{not college}) = \frac{P(\text{f} \cap \text{not college})}{P(\text{not college})} = \frac{0.04}{0.04 + 0.06} = \frac{0.04}{0.1} = 0.40
\]
Yen Diagrams

1. If for your birthday, P(you getting an iPod) = 0.66, P(you getting a new car) = 0.54, and P(getting both) = 0.23, find P(you getting a new car OR iPod).

\[
P(C \text{ or } I) = P(C) + P(I) - P(C \cap I) = 0.54 + 0.66 - 0.23 = 0.97
\]

2. Randomly selected high school students were surveyed about their caffeine habits, as shown.

- 55% drink coffee
- 25% drink tea
- 43% drink soda
- 15% drink coffee and tea
- 25% drink coffee and soda
- 5% drink only tea
- 5% drink all three

\[P(\text{only soda}) = 0.15\]

\[P(\text{not caffeine}) = 1 - (0.55 + 0.25) = 0.20\]

M&M Explorations

<table>
<thead>
<tr>
<th></th>
<th>Red</th>
<th>Orange</th>
<th>Yellow</th>
<th>Green</th>
<th>Blue</th>
<th>Brown</th>
<th>TOTAL</th>
</tr>
</thead>
<tbody>
<tr>
<td>Peanut</td>
<td>3</td>
<td>6</td>
<td>4</td>
<td>4</td>
<td>6</td>
<td>2</td>
<td>25</td>
</tr>
<tr>
<td>Plain</td>
<td>10</td>
<td>15</td>
<td>11</td>
<td>12</td>
<td>18</td>
<td>9</td>
<td>75</td>
</tr>
<tr>
<td>TOTAL</td>
<td>13</td>
<td>21</td>
<td>15</td>
<td>16</td>
<td>24</td>
<td>11</td>
<td>100</td>
</tr>
</tbody>
</table>

Here, "other" = other than red

- \(P(\text{red } \cap \text{ peanut})\)
- \(P(\text{red } \cap \text{ plain})\)
- \(P(\text{other } \cap \text{ peanut})\)
- \(P(\text{other } \cap \text{ plain})\)

a) What is the probability that a randomly selected M&M will be peanut?

b) What is the probability that a plain M&M is other?

c) Given that an M&M is plain, what is the probability that this M&M is red?
Probability Worksheet – Setting up Tables

1. A company’s records indicate that on any given day about 1% of their day shift employees and 2% of the night shift employees will miss work. 60% of the employees work the day shift.
   a. Is absenteeism independent of the shift worked? Explain.

<table>
<thead>
<tr>
<th></th>
<th>Day</th>
<th>Night</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>Miss</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>~ Miss</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Total</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

   If independent, 
   \[
P (\text{miss & day}) = P (\text{miss}) \cdot P (\text{day}) \quad \text{and} \quad P (\text{miss & night}) = P (\text{miss}) \cdot P (\text{night})
   \]

   Conclusion:

   b. What per cent of the employees are absent on any given day?

   c. What per cent of the absent employees are on the night shift?

2. Police often set up sobriety checkpoints to judge whether a person may have been drinking. If the officer does not suspect a problem, drivers are released. Otherwise, drivers are detained for a Breathalyzer test to determine whether or not they are arrested. Based on the brief initial stop, trained officers can make the right decision 80% of the time. Suppose the police operate a sobriety checkpoint after 9 p.m. on a Saturday night, a time when national traffic safety experts suspect that about 12% of drivers have been drinking.

<table>
<thead>
<tr>
<th></th>
<th>Detained</th>
<th>Not Detained</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>Drinking</td>
<td>.096</td>
<td>.024</td>
<td>.120</td>
</tr>
<tr>
<td>~ Drinking</td>
<td>.176</td>
<td>.704</td>
<td>.880</td>
</tr>
<tr>
<td>Total</td>
<td>.272</td>
<td>.728</td>
<td>1.000</td>
</tr>
</tbody>
</table>

   a. You are stopped at the checkpoint and, of course, have not been drinking. What is the probability that you are detained for further testing?
   \[
   P (D | \overline{D}) = \frac{P (D \overline{D})}{P (D)} = \frac{.024}{.072} = .333
   \]

   b. What is the probability that any given driver will be detained?

   c. What is the probability that a driver who is detained has actually been drinking?

   \[
   P (D | R) = \frac{P (D | R)}{P (R)} = \frac{.024}{.0728} = .0329
   \]

   d. What is the probability that a driver who was released actually been drinking?
Conditional Probability

1. Refer to the table below.

<table>
<thead>
<tr>
<th>Age</th>
<th>&lt;$20,000</th>
<th>$20,000–$50,000</th>
<th>&gt;$50,000</th>
</tr>
</thead>
<tbody>
<tr>
<td>&lt;25</td>
<td>950</td>
<td>1,000</td>
<td>50</td>
</tr>
<tr>
<td>25–45</td>
<td>450</td>
<td>2,050</td>
<td>1,500</td>
</tr>
<tr>
<td>&gt;45</td>
<td>50</td>
<td>950</td>
<td>1,000</td>
</tr>
</tbody>
</table>

A: \{\text{Person is under 25}\}
B: \{\text{Person is between 25 and 45}\}
C: \{\text{Person is over 45}\}
D: \{\text{Person has income under $20,000}\}
E: \{\text{Person has income of $20,000 - $50,000}\}
F: \{\text{Person has income over $50,000}\}

Find each of the following probabilities (do not round off the decimal).

a. \( P(B) \)

b. \( P(F) \)

c. \( P(C \cap F) \)

d. \( P(B \cup C) \)

e. \( P(\sim A) \)

f. \( P(\sim A \cap F) \)
Conditional Probability

1. In “Crime, race and reporting to the police,” R. Shah and K. Pease examined the relationship between the race of the attacker, the race of the victim, and the degree of injury sustained in reported crimes in a British crime survey (The Howard Journal of Criminal Justice, Aug. 1992). The table below is cited in the article.

<table>
<thead>
<tr>
<th>Degree of Injury</th>
<th>White/White</th>
<th>White/Nonwhite</th>
<th>Nonwhite/Nonwhite</th>
<th>Nonwhite/White</th>
<th>Totals</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fatal</td>
<td>183</td>
<td>18</td>
<td>18</td>
<td>18</td>
<td>237</td>
</tr>
<tr>
<td>Serious</td>
<td>580</td>
<td>49</td>
<td>111</td>
<td>141</td>
<td>881</td>
</tr>
<tr>
<td>Slight</td>
<td>4,336</td>
<td>440</td>
<td>594</td>
<td>1,656</td>
<td>7,026</td>
</tr>
<tr>
<td>None</td>
<td>1,422</td>
<td>136</td>
<td>214</td>
<td>1,801</td>
<td>3,573</td>
</tr>
<tr>
<td>Totals</td>
<td>6,521</td>
<td>643</td>
<td>937</td>
<td>3,616</td>
<td>11,717</td>
</tr>
</tbody>
</table>


a. What is the probability that a randomly selected reported crime involved a white attacker and a white victim?

b. What is the probability that a randomly selected reported crime involved serious injuries?

c. Given that both the attacker and victim were white, what is the probability that a randomly selected reported crime involved fatalities?

d. If no injury was reported, what is the probability that a randomly selected reported crime involved a nonwhite attacker and a white victim?
2. The *American Journal of Public Health* published a study of unintentional carbon monoxide (CO) poisonings in Colorado, where 981 cases were classified in the table below. A case of unintentional COP poisoning is chosen at random from the 981 cases.

<table>
<thead>
<tr>
<th>Source of Exposure</th>
<th>Fatal</th>
<th>Nonfatal</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fire</td>
<td>63</td>
<td>53</td>
<td>116</td>
</tr>
<tr>
<td>Auto exhaust</td>
<td>60</td>
<td>178</td>
<td>238</td>
</tr>
<tr>
<td>Furnace</td>
<td>18</td>
<td>345</td>
<td>363</td>
</tr>
<tr>
<td>Kerosene or space heater</td>
<td>9</td>
<td>18</td>
<td>27</td>
</tr>
<tr>
<td>Appliance</td>
<td>9</td>
<td>63</td>
<td>72</td>
</tr>
<tr>
<td>Other gas-powered motor</td>
<td>3</td>
<td>73</td>
<td>76</td>
</tr>
<tr>
<td>Fireplace</td>
<td>0</td>
<td>16</td>
<td>16</td>
</tr>
<tr>
<td>Other</td>
<td>3</td>
<td>19</td>
<td>22</td>
</tr>
<tr>
<td>Unknown</td>
<td>9</td>
<td>42</td>
<td>51</td>
</tr>
<tr>
<td><strong>Total</strong></td>
<td>174</td>
<td>807</td>
<td>981</td>
</tr>
</tbody>
</table>


a. Given that the source of the poisoning is fire, what is the probability that the case is fatal?

b. Given that the case is nonfatal, what is the probability that it is caused by auto exhaust?

c. If the case is fatal, what is the probability that the source is unknown?

d. If the case is nonfatal, what is the probability that the source is not fire or a fireplace?
Conditional Probability

3. There are several methods of typing, or classifying human blood. The most common procedure types blood into the general classifications of A, B, O, or AB. A method that is not as well known examines phosphoglucomutase (PGM) and classifies the blood into one of three main categories, 1-1, 2-1, or 2-2. Suppose a certain geographic region of the U.S. has the PGM percentages shown in the table below. A person is to be chosen at random from this region.

<table>
<thead>
<tr>
<th>Race</th>
<th>1-1</th>
<th>2-1</th>
<th>2-2</th>
</tr>
</thead>
<tbody>
<tr>
<td>White</td>
<td>46.3%</td>
<td>39.2%</td>
<td>4.0%</td>
</tr>
<tr>
<td>Black</td>
<td>6.7%</td>
<td>3.4%</td>
<td>.4%</td>
</tr>
</tbody>
</table>

a. What is the probability that a black person is chosen?

b. Given that a black person is chosen, what is the probability that he/she is PGM type 1-1?

c. Given that a white person is chosen, what is the probability that he/she is PGM type 1-1?

4. “Channel 1” is an educational television network available to all secondary schools in the U.S. According to Educational Technology (May-June 1995), 40% of all U.S. secondary schools subscribe to Channel One Communications Network (CNN). Of these subscribers, only 5% never use the CNN broadcasts, while 20% use CNN more than five times per week.

a. Find the probability that a randomly selected U.S. secondary school subscribes to CNN but never uses the CNN broadcasts.

b. Find the probability that a randomly selected U.S. secondary school subscribes to CNN and uses the broadcasts more than five times per week.