Chapter 21 More About Tests

27. Dropouts. A statistics professor has observed that for several years about 13% of the students who initially enroll in his Introductory Statistics course withdraw before the end of the semester. A salesman suggests that he try a statistics software package that gets students more involved with computers, predicting that it will cut the dropout rate. The software is expensive, and the salesman offers to let the professor use it for a semester to see if the dropout rate goes down significantly. The professor will have to pay for the software only if he chooses to continue using it.

a. Is this a one-tailed or two-tailed test? Explain.

b. Write the null and alternative hypotheses.

\[ H_0: p = 0.13 \]
\[ H_A: p < 0.13 \]
c. In this context, explain what would happen if the professor makes a Type I error.

d. In this context, explain what would happen if the professor makes a Type II error.

e. What is meant by the power of the test?
   a) Type II. The filter decided that the message was safe, when in fact it was spam.
   b) Type I. The filter decided that the message was spam, when in fact it was not.
   c) This is analogous to lowering alpha. It takes more evidence to classify a message as spam.
   d) The risk of Type I error is decreased and the risk of Type II error has increased.

   a) The power of the test is the ability of the filter to detect spam.
   b) To increase the filter’s power, lower the cutoff score.
   c) If the cutoff score is lowered, a larger number of real messages would end up in the junk mailbox.

29. Dropouts, part II.
   a) \( H_0: \) The dropout rate does not change following the use of the software. \( p = 0.13 \)
   \( H_A: \) The dropout rate decreases following the use of the software. \( p < 0.13 \)

   **Independence assumption:** One student’s decision about dropping out should not influence another’s decision.

   **Randomization condition:** This year’s class of 203 students is probably representative of all students.

   **10% condition:** A sample of 203 students is less than 10% of all students.

   **Success/Failure condition:** \( np = (203)(0.13) = 26.39 \) and \( nq = (203)(0.87) = 176.61 \) are both greater than 10, so the sample is large enough.

   The conditions have been satisfied, so a Normal model can be used to model the sampling distribution of the proportion, with \( \mu_p = p = 0.13 \) and \( \sigma(\hat{p}) = \sqrt{\frac{pq}{n}} = \sqrt{\frac{(0.13)(0.87)}{203}} \approx 0.0236. \)

   We can perform a one-proportion z-test. The observed proportion of dropouts is \( \hat{p} = \frac{11}{203} \approx 0.054. \)

   Since the P-value = 0.0007 is very low, we reject the null hypothesis. There is strong evidence that the dropout rate has dropped since use of the software program was implemented. As long as the professor feels confident that this class of students is representative of all potential students, then he should buy the program.

   If you used a 95% confidence interval to assess the effectiveness of the program:
   \[
   \hat{p} \pm z^* \sqrt{\frac{pq}{n}} = \left( \frac{11}{203} \right) \pm 1.960 \sqrt{\frac{\left( \frac{11}{203} \right) \left( \frac{192}{203} \right)}{203}} = (2.3\%, 8.5\%)
   \]

   We are 95% confident that the dropout rate is between 2.3% and 8.5%. Since 15% is not contained in the interval, this provides evidence that the dropout rate has changed following the implementation of the software program.

   b) The chance of observing 11 or fewer dropouts in a class of 203 is only 0.07% if the dropout rate in the population is really 13%.
30. Testing the ads.

a) $H_0$: The percentage of residents that remember the ad is 20%. ($p = 0.20$)
$H_A$: The percentage of residents that remember the ad is greater than 20%. ($p > 0.20$)

**Independence assumption:** It is reasonable to think that randomly selected residents would remember the ad independently of one another.

**Randomization condition:** The sample consisted of 600 randomly selected residents.

**10% condition:** The sample of 600 is less than 10% of the population of the city.

**Success/Failure condition:** $np = (600)(0.20) = 120$ and $nq = (600)(0.80) = 480$ are both greater than 10, so the sample is large enough.

The conditions have been satisfied, so a Normal model can be used to model the sampling distribution of the proportion, with $\mu_p = p = 0.20$ and $\sigma(\hat{p}) = \sqrt{\frac{pq}{n}} = \sqrt{\frac{(0.20)(0.80)}{600}} = 0.0163$.

We can perform a one-proportion $z$-test. The observed proportion of residents who remembered the ad is $\hat{p} = \frac{133}{600} = 0.222$.

Since the $P$-value = 0.0923 is somewhat high, we fail to reject the null hypothesis. There is little evidence that more than 20% of people remember the ad. The company should not renew the contract.

b) There is a 9.23% chance of having 133 or fewer of 600 people in a random sample remember the ad, if in fact only 20% of people in the population do.