A Brief History of the Pythagorean Theorem

Just Who Was This Pythagoras, Anyway?

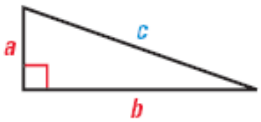
Pythagoras <http://www-groups.dcs.st-and.ac.uk/~history/BigPictures/Pythagoras.jpeg>(569-500 B.C.E.) was born on the island of Samos in Greece, and did much traveling through Egypt, learning, among other things, mathematics. Not much more is known of his early years. Pythagoras gained his famous status by founding a group, the Brotherhood of Pythagoreans, which was devoted to the study of mathematics. The group was almost cult-like in that it had symbols, rituals and prayers. In addition, Pythagoras believed that "Number rules the universe,"and the Pythagoreans gave numerical values to many objects and ideas. These numerical values, in turn, were endowed with mystical and spiritual qualities.

Legend has it that upon completion of his famous theorem, Pythagoras sacrificed 100 oxen. Although he is credited with the discovery of the famous theorem, it is not possible to tell if Pythagoras is the actual author. The Pythagoreans wrote many geometric proofs, but it is difficult to ascertain who proved what, as the group wanted to keep their findings secret. Unfortunately, this vow of secrecy prevented an important mathematical idea from being made public. The Pythagoreans had discovered irrational numbers! If we take an isosceles right triangle with legs of measure 1, the hypotenuse will measure sqrt 2. But this number cannot be expressed as a length that can be measured with a ruler divided into fractional parts, and that deeply disturbed the Pythagoreans, who believed that "All is number." They called these numbers "alogon," which means "unutterable." So shocked were the Pythagoreans by these numbers, they put to death a member who dared to mention their existence to the public. It would be 200 years later that the Greek mathematician Eudoxus <http://www-groups.dcs.st-and.ac.uk/~history/Mathematicians/Eudoxus.html> developed a way to deal with these unutterable numbers.

8.2 - The Pythagorean Theorem and its Converse

The Pythagorean Theorem:

In a right triangle, the square of the length of the hypotenuse is equal to the sum of the squares of the lengths of the legs.

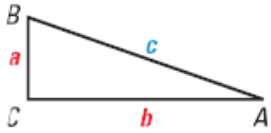


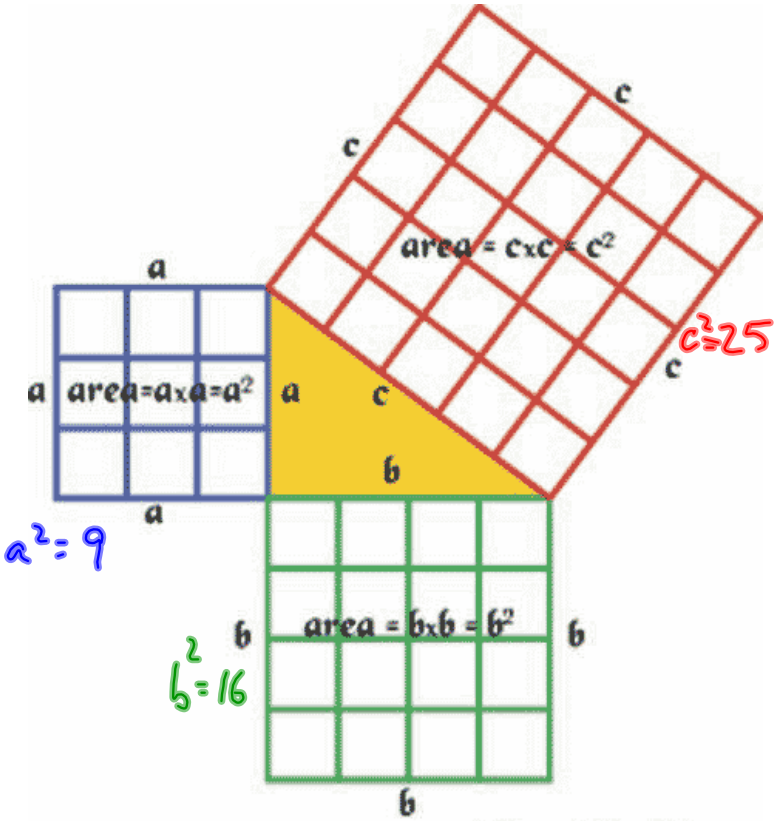
$$a^2 + b^2 = c^2$$

The Converse of the Pythagorean Theorem:

If the square of the length of the longest side of a triangle is equal to the sum of the squares of the lengths of the other two sides, then the triangle is a right triangle.

If $c^2 = a^2 + b^2$, then $\triangle ABC$ is a right triangle.





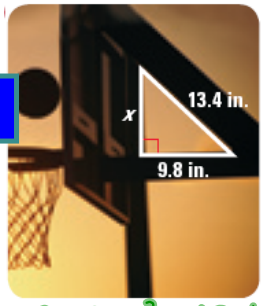
Pythagoream Theorem: $c^2 = a^2 + b^2$

Sample Problems (using the Pythagor[redacted]heorem):

$x^2 + 8.9^2 = 16.7^2$

OR $16.7^2 - 8.9^2 = x^2$

[redacted]



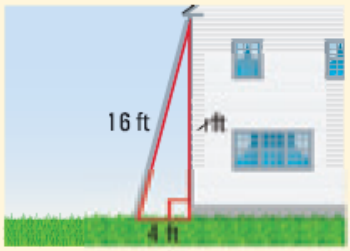
$x^2 + 9.8^2 = 13.4^2$

$x^2 + 4.9^2 = 5.7^2$

A 16 foot ladder rests against the side of the house, and the base of the ladder is 4 feet away. Approximately how high above the ground is the top of the ladder?

(A) ~~240 feet~~ (B) ~~20 feet~~

(C) ~~16.5 feet~~ (D) 15.5 feet

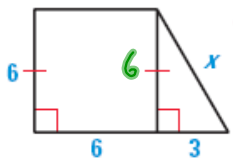


$4^2 + x^2 = 16^2$

[redacted]

FINDING SIDE LENGTHS Find the unknown side length x. Write your answer in simplest radical form.

24. $3\sqrt{5}$

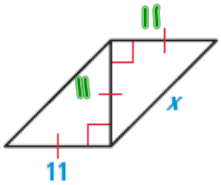


$6^2 + 3^2 = x^2$

$\sqrt{45} = x$

$3\sqrt{5} = x$

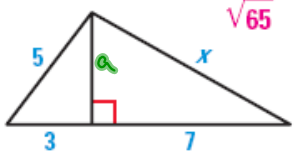
25. $11\sqrt{2}$



$x = 11\sqrt{2}$

$11^2 + 11^2 = x^2$

26. $\sqrt{65}$



$3^2 + a^2 = 5^2$

$a = 4$

$4^2 + 7^2 = x^2$

$\sqrt{65} = x$

Pythagorean Triples:

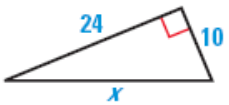
Common Pythagorean Triples and Some of Their Multiples

3, 4, 5	**5, 12, 13**	**8, 15, 17**	**7, 24, 25**
6, 8, 10	10, 24, 26	16, 30, 34	14, 48, 50
9, 12, 15	15, 36, 39	24, 45, 51	21, 72, 75
30, 40, 50	50, 120, 130	80, 150, 170	70, 240, 250
$3x$, $4x$, $5x$	$5x$, $12x$, $13x$	$8x$, $15x$, $17x$	$7x$, $24x$, $25x$

The most common Pythagorean triples are in bold. The other triples are the result of multiplying each integer in a bold face triple by the same factor.

Sample Problem:

Find the length of the hypotenuse of the right triangle.



Solution

Method 1: Use a Pythagorean triple.

A common Pythagorean triple is 5, 12, 13. Notice that if you multiply the lengths of the legs of the Pythagorean triple by 2, you get the lengths of the legs of this triangle: $5 \cdot 2 = 10$ and $12 \cdot 2 = 24$. So, the length of the hypotenuse is $13 \cdot 2 = 26$.

Method 2: Use the Pythagorean Theorem.

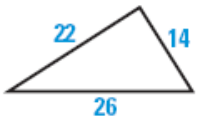
$x^2 = 10^2 + 24^2$	Pythagorean Theorem
$x^2 = 100 + 576$	Multiply.
$x^2 = 676$	Add.
$x = 26$	Find the positive square root.

Sample Problems (using the Converse of Pythagorean Theorem):

Tell whether the given triangle is a right triangle.

a.

b.



Let c represent the length of the longest side of the triangle. Check to see whether the side lengths satisfy the equation $c^2 = a^2 + b^2$.

a. $(3\sqrt{34})^2 \stackrel{?}{=} 9^2 + 15^2$

$9 \cdot 34 \stackrel{?}{=} 81 + 225$

$306 = 306$ ✓

The triangle is a right triangle.

b. $26^2 \stackrel{?}{=} 22^2 + 14^2$

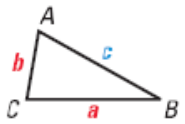
$676 \stackrel{?}{=} 484 + 196$

$676 \neq 680$

The triangle is not a right triangle.

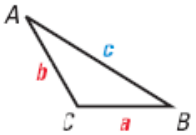
Classifying Other Triangles:

If the square of the length of the longest side of a triangle is less than the sum of the squares of the lengths of the other two sides, then the triangle ABC is an acute triangle.



If $c^2 < a^2 + b^2$, then the triangle ABC is acute.

If the square of the length of the longest side of a triangle is greater than the sum of the squares of the lengths of the other two sides, then the triangle ABC is an obtuse triangle.



If $c^2 > a^2 + b^2$, then triangle ABC is obtuse.

WALKING You walk 749 feet due east to the gym from your home. From the gym you walk 800 feet southwest to the library. Finally, you walk 305 feet from the library back home. Do you live directly north of the library? *Explain.*

Class Work and Homework for Friday Night:

pg. 292: 1-18 (omit #12) (all)
and pg. 292,293: 19-29 (odds)
and pg. 297: 1-10

Draw pics and show work!